A metamaterial slab as a lens, a cloak and something in between

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Outline

- Is a metamaterial slab a lens or a cloak ?
  - Lens: Device that enable you to see an object
  - Cloak: Device to hide an object
- Answer:
  - It can be a lens, It can be a cloak
  - In most of the cases, it is somewhere in between

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Pendry, Schurig and Smith, Science, (2006)

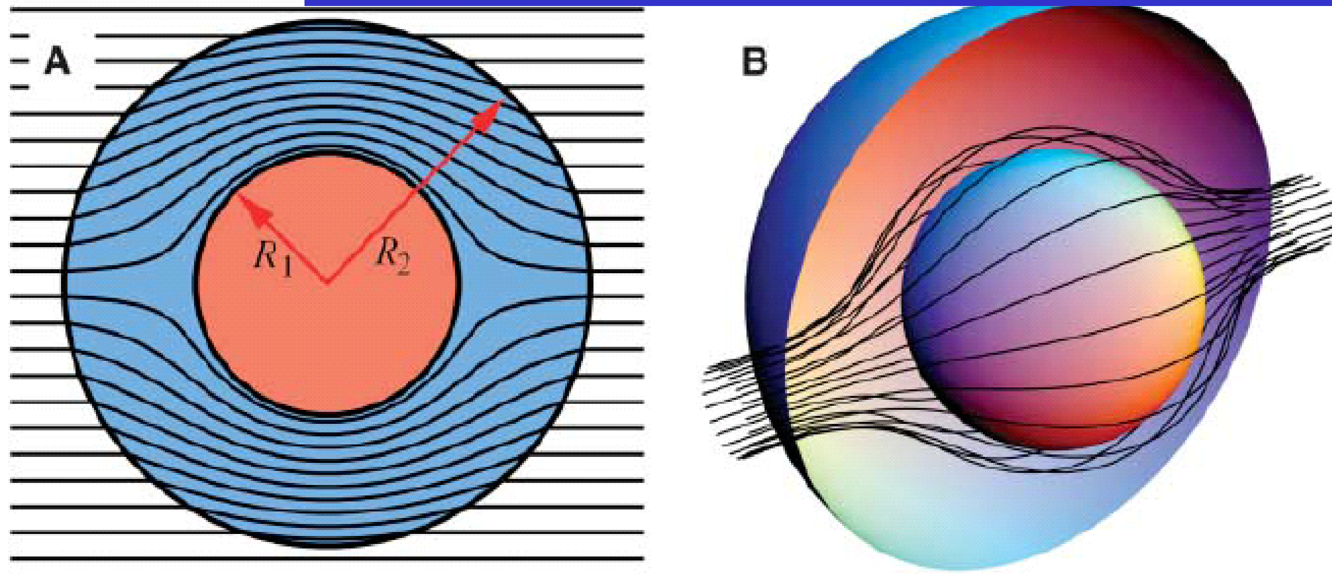


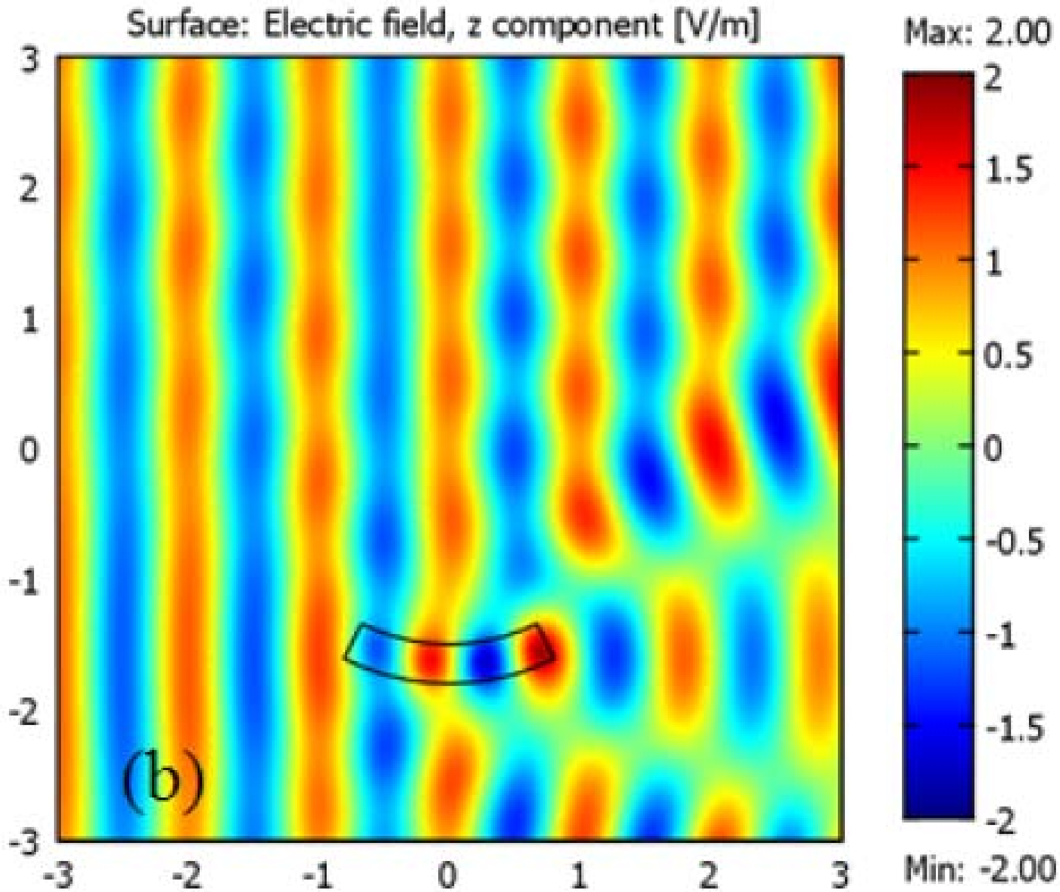
Fig. 2. A ray-tracing program has been used to calculate ray trajectories in the cloak, assuming that $R_2 \gg \lambda$. The rays essentially following the Poynting vector. (**A**) A two-dimensional (2D) cross section of rays striking our system, diverted within the annulus of cloaking material contained within $R_1 < r < R_2$ to emerge on the far side undeviated from their original course. (**B**) A 3D view of the same process.

Positive index metamaterial guides light "around" an enclosed domain:

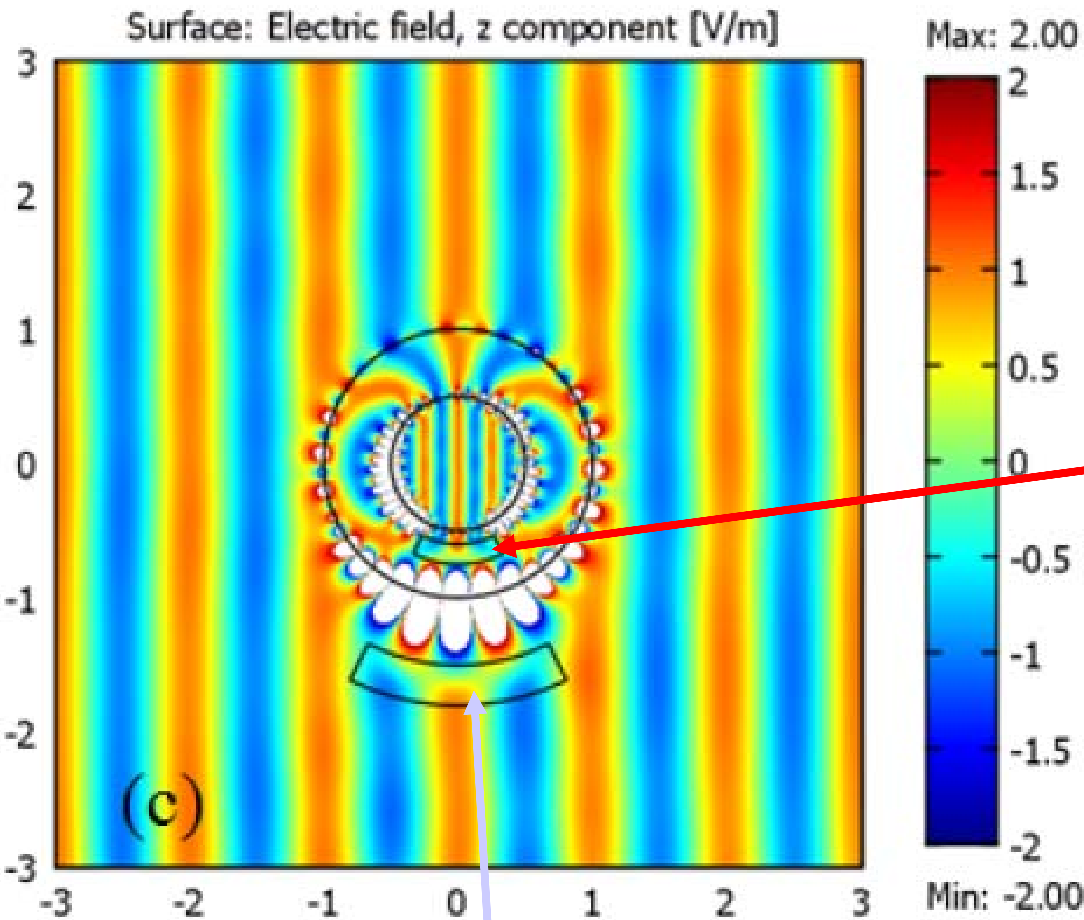
- Cloaks anything inside that cloaking shell
- The cloaked object is surrounded by the cloak

Can we do cloaking at a distance: Cloak does not encircle object ?

Do "external cloaking" to hide this object:



“anti-object” inside negative index shell: cancels scattering of the object



Anti – object:

$$\frac{\mu'_{or}}{\mu_{or}} = \frac{f(r')}{r'} \frac{1}{f'(r')}$$

$$\frac{\mu'_{o\theta}}{\mu_{o\theta}} = \frac{r'}{f(r')} f'(r')$$

$$\frac{\varepsilon'_{oz}}{\varepsilon_{oz}} = \frac{f(r')}{r'} f'(r')$$

object outside the cloak, within b<r<c

Phys. Rev. Lett. 102, 093901 (2009).

Generalization: Illusion Optics

- Invisibility cloak is a special case of illusion
  - An object is "optically transformed" by an device into "free space", and thereby becomes invisible.
- Can we have an device that "optically transform" one object into another object?

Transformation Optics ?

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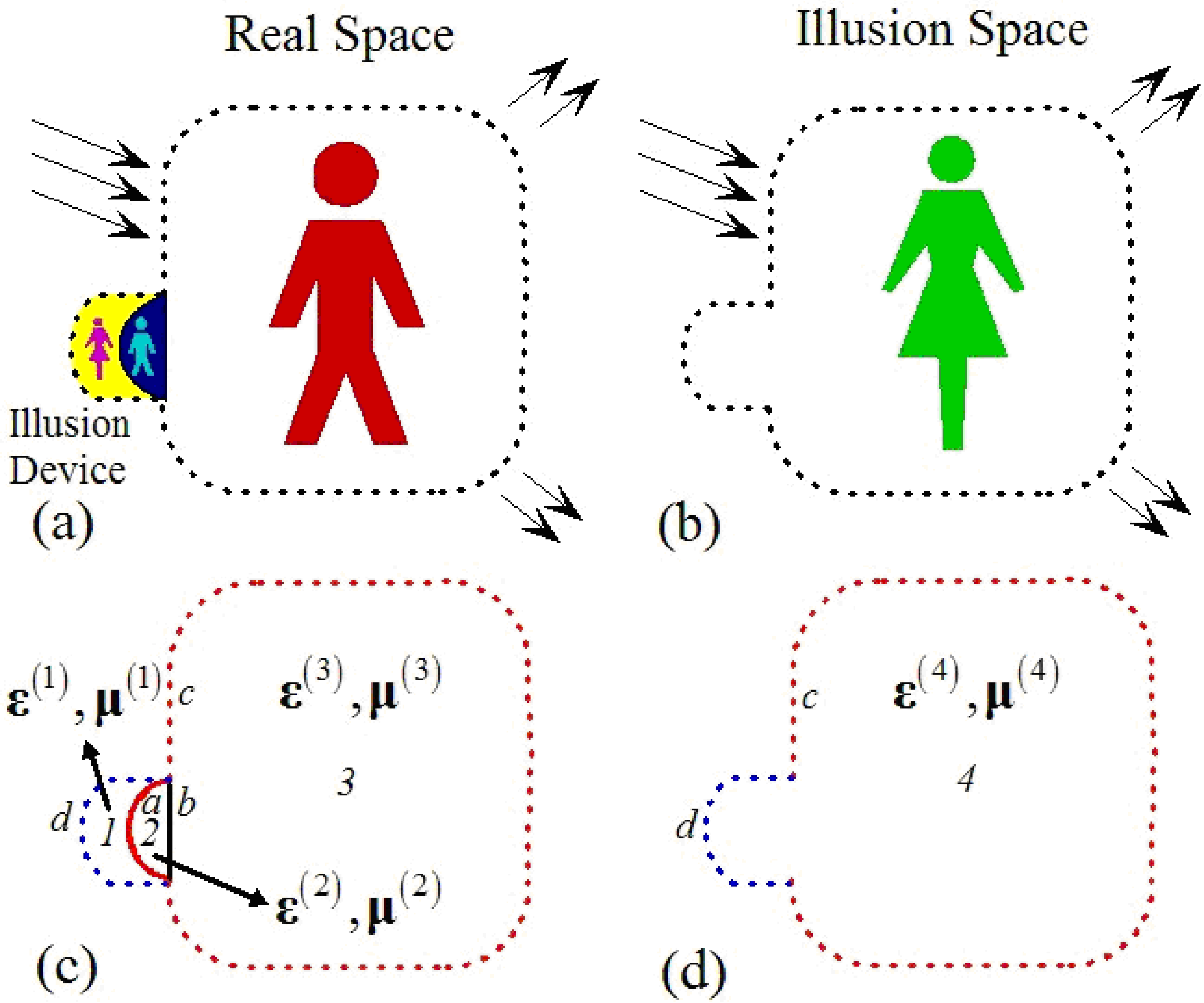
ion Optics

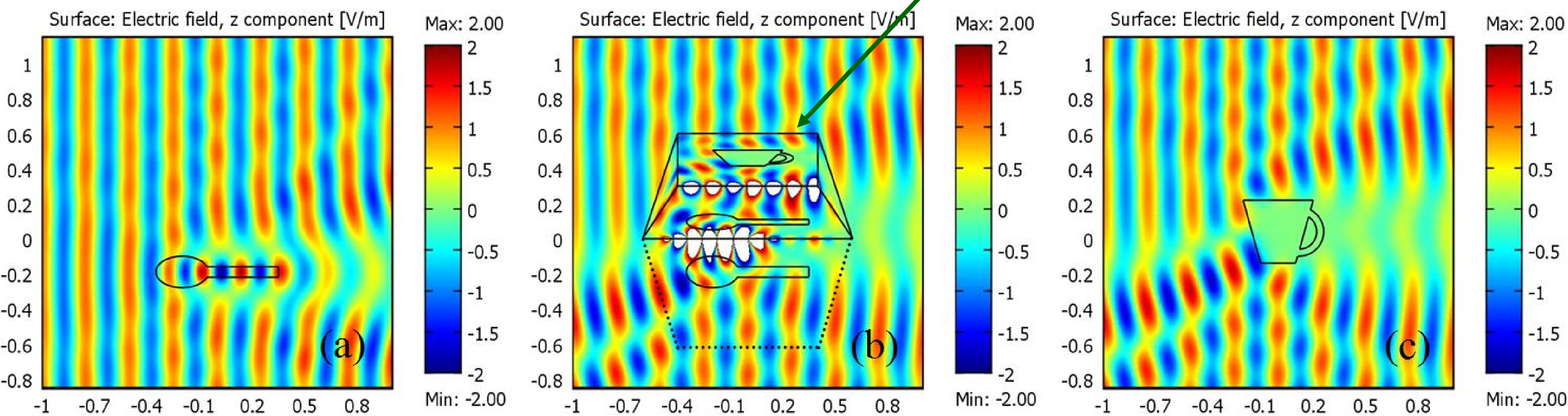


- Two steps
  - Optically cancel the man
  - Optically project the woman

Phys. Rev. Lett. 102, 253902 (2009)

Changing a spoon into a cup

Device with specific electromagnetic properties defined by transformation media



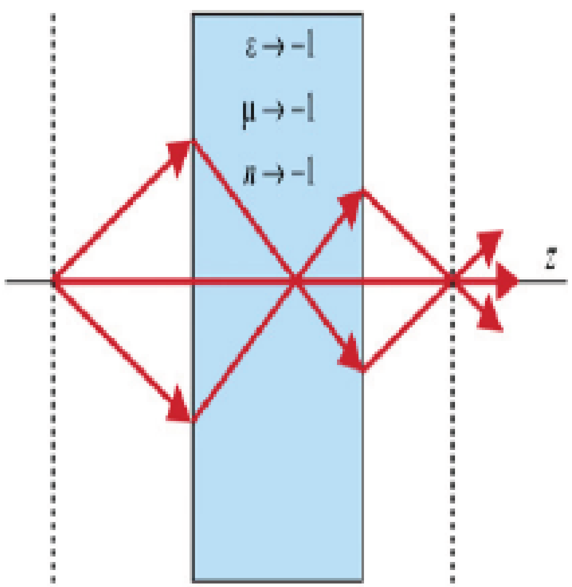
The scattering pattern of a dielectric spoon

The scattering pattern of the dielectric spoon is changed by an illusion device into that of a metallic cup

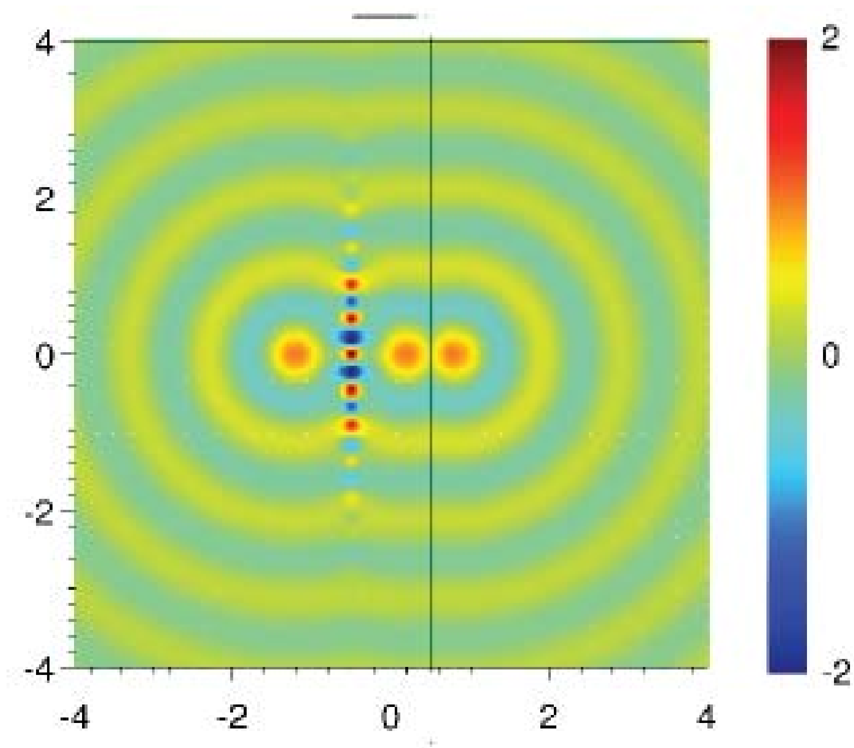
The scattering pattern of a metallic cup

PRL. 102, 253902 (2009)

J. Pendry (2000): ε=μ= -1 slab is a perfect lens (image has perfect resolution)



Perfect lens. Source-image distance: 2d.



Imaging a point source

G. Milton et al

- A collection of point dipoles is cloaked by ε=-1 shell in the quasi-static limit

$$\varepsilon_{core} = \varepsilon_{medium} = 1$$

$$\varepsilon_{shell} = -1 + (small)i$$

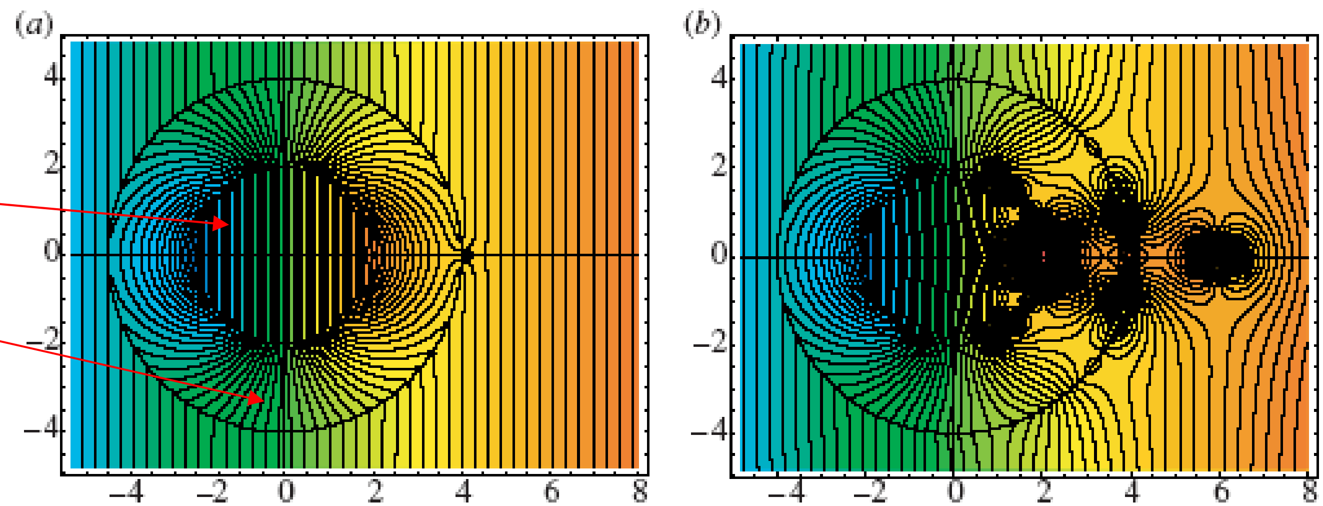


Figure 3. Numerical computations of equipotentials for $\mathrm{Re}[V(z)]$ for a cylindrical lens with $\varepsilon_m = \varepsilon_c = 1$ and $\varepsilon_s = -1 + 0.01i$, and with $r_c = 2$, $r_s = 4$, $r_* = 8$, and $r_\# = 5.66$. A uniform field $\boldsymbol{E} = (-1, 0)$ acts on the system. In the figure on the left the polarizable line dipole with $\alpha = 2$ and $(k_0^e, k_0^o) = 0$ is located close to the lens at $r_0 = 4.166$ and it along with the cylindrical lens are essentially invisible to the uniform field. The computations show that the polarizable line dipole has $k^e = 0.000012 - 0.00066i$ and $k^o = 0$. In the figure on the right the polarizable line dipole is located outside the cloaking region at $r_0 = 5.95$ and significantly perturbs the uniform field. The computations show that the polarizable line dipole has $k^e = -1.68 - 0.74i$ and $k^o = 0$.

Milton and Nicorovici, Proc. R. Soc. A (2006) 462, 3027

Nicorovici, McPhedran, Milton, Phys. Rev. B 49, 8479 (1994).

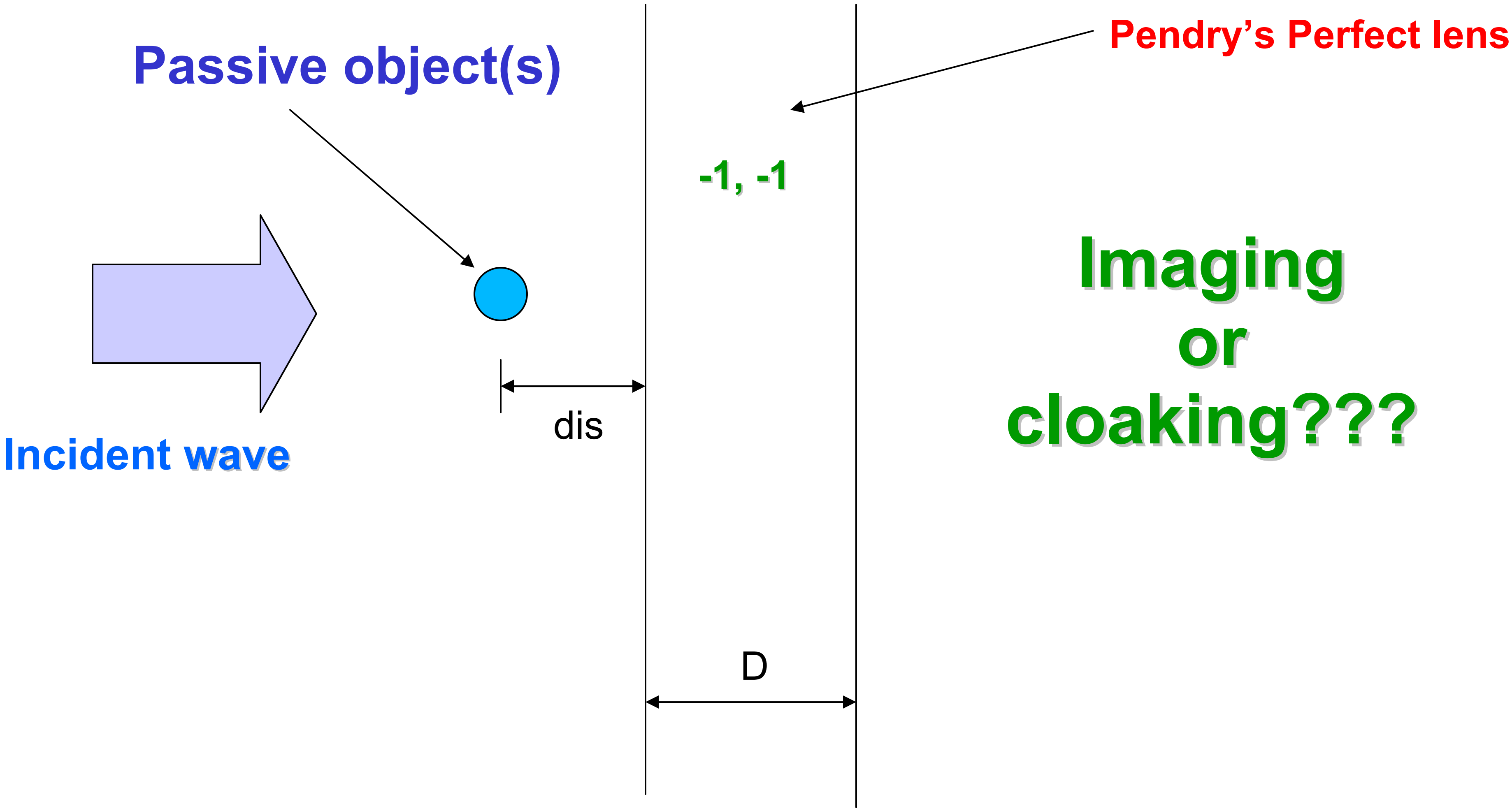
Simple Extension of Milton's result

- Milton et al. results also holds in finite frequency:
- $\varepsilon=\mu=-1$ slab is a perfect cloak for small objects
- A collection of small objects will be invisible if it is placed close to the $\varepsilon=\mu=-1$ slab

Contradiction ? Is ($\varepsilon=\mu= -1$) a lens or a cloak ?

- Pendry considers an active source
- Milton *et al* consider an passive object illuminated from elsewhere
- We will consider
  - the object being imaged is “passive”
  - always consider a finite absorption, which can be made as small as we please

Perfect Lens

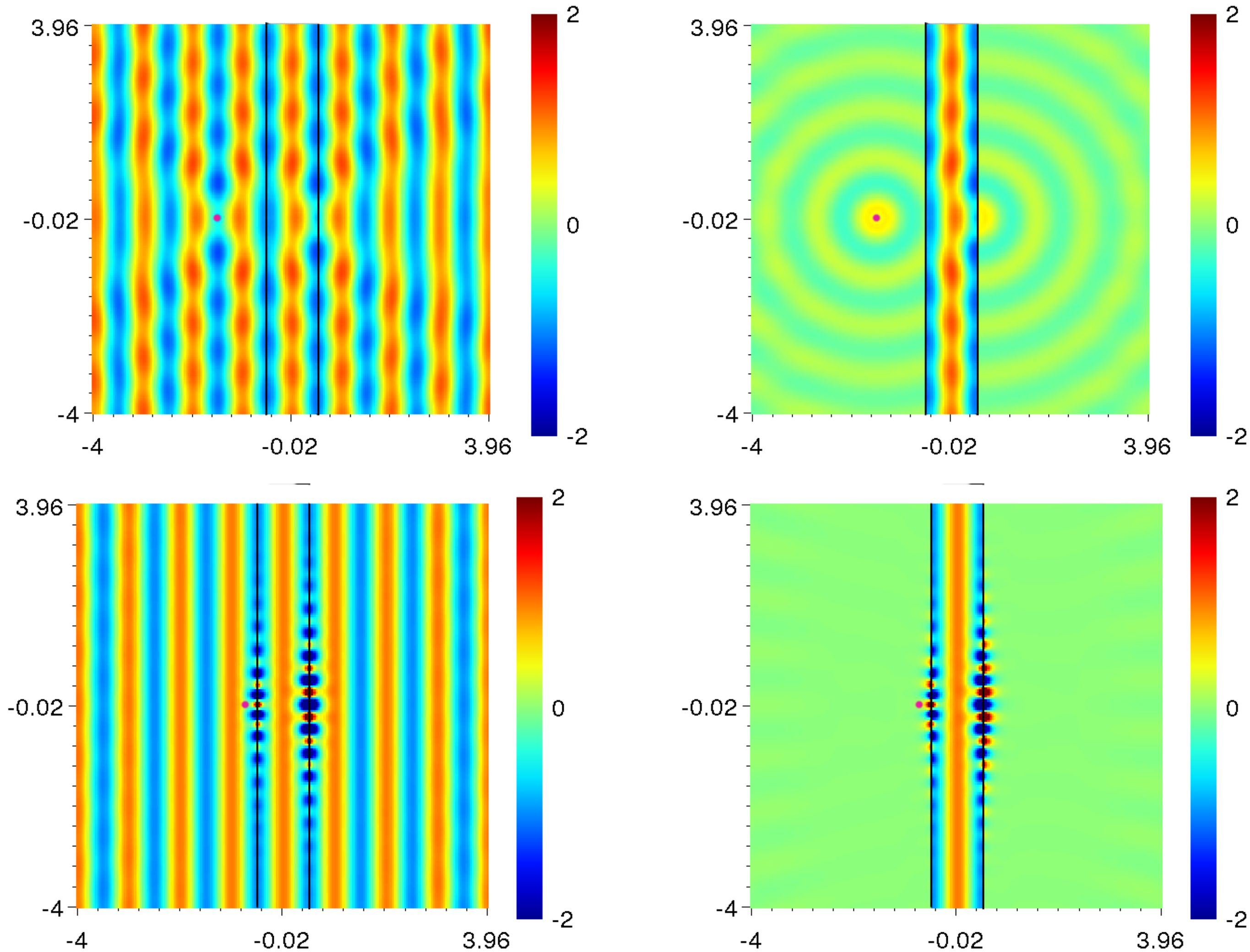


G W. Milton and N P. Nicorovici, Proc. R. Soc. A (2006) **462**, 3027–3059
J. B. Pendry, Phys. Rev. Lett. **85**, 3966(2000)

Perfect Lens: Imaging or Cloaking

distance > ½ D
Imaging

distance < ½ D
Cloaking

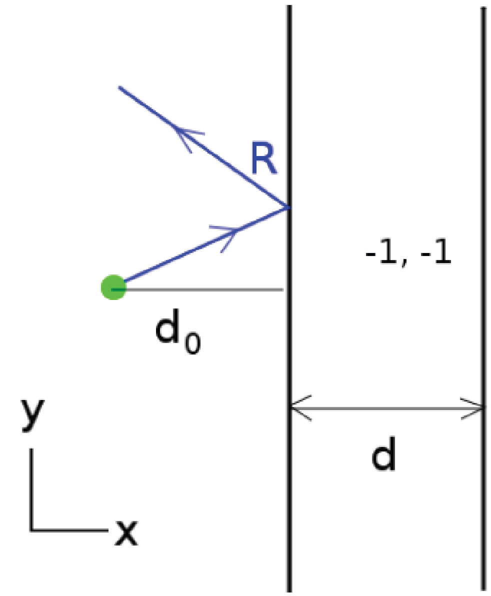


Total field **Scattering field**

Cylinder:
radius = 0.005,
ε = 400
λ = 1.0
Slab: D = 1.0

Numerical simulation using BEM

Cloaking for a monopole: 2D



Response of a monopole $pH_0(kr)$

$$p = \alpha(\phi^{\text{inc}} + \phi^{\text{ref}}) \,, \tag{3.1}$$

where α is the polarizability, ϕ^{inc} is the external incident field, and ϕ^{ref} is the reflected field.

$\phi^{\text{ref}} = 0$: imaging

$\phi^{\text{ref}} = -\phi^{\text{inc}}$: cloaking

ϕ^{ref} is proportional to p.

$$p = \alpha\left\{\phi^{\text{inc}} + p\int_{-\infty}^{\infty} R(k_{\|})\exp[2ik_x d_0] f(k_{\|}) dk_{\|}\right\},$$

$$k_{\|}^2 + k_x^2 = k_0^2\,, \tag{3.2}$$

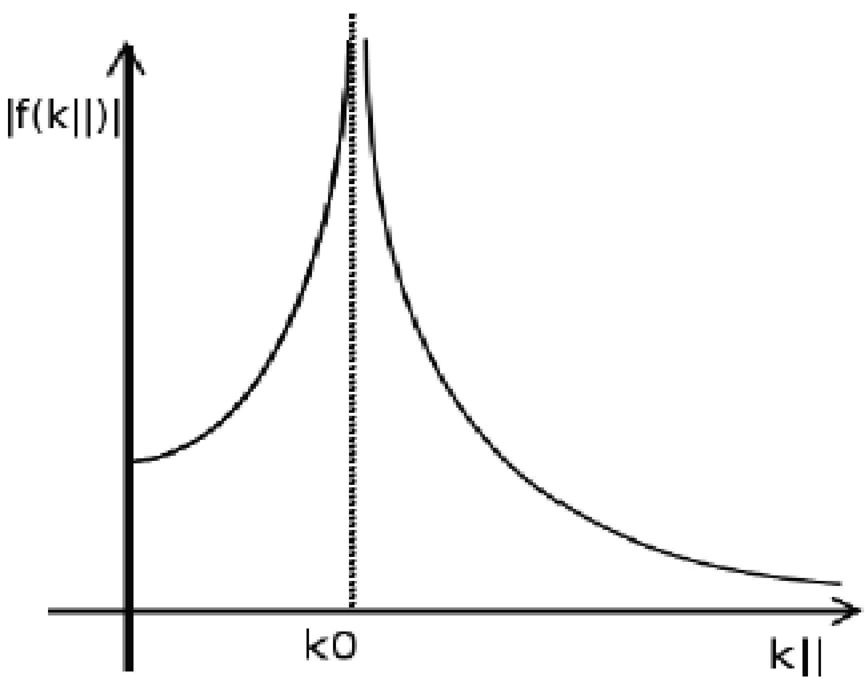
$R(k_{\|})$ is the reflection coefficient of the slab, $f(k_{\|})$ is the Fourier spectrum of a monopole, and d_0 is the distance from the monopole to the slab interface.

$$p = \alpha\phi^{\text{inc}}/(1-\alpha\mathcal{I}), \quad \mathcal{I} = \int_{-\infty}^{\infty} R(k_{\|})\exp[2ik_x d_0] f(k_{\|}) dk_{\|}$$

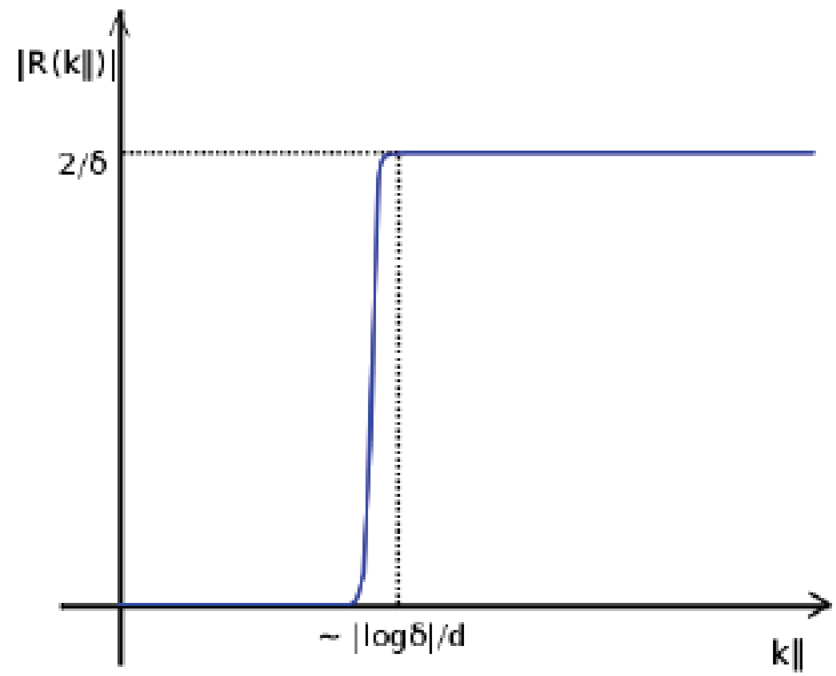
$$f(k_{\|}) = i/2\sqrt{k_0^2 - k_{\|}^2}, \quad R(k_{\|}) = \frac{4\delta\sin[ik_x' d]}{\delta^2\exp[-ik_x' d] + 4\exp[ik_x' d]}$$

$$p = \alpha\phi^{\text{inc}}/(1-\alpha\mathcal{I}), \quad \mathcal{I} = \int_{-\infty} R(k_{\parallel})\exp[2ik_x d_0] f(k_{\parallel}) dk_{\parallel}$$

$$f(k_{\parallel}) = i/2\sqrt{k_0^2 - k_{\parallel}^2}, \quad R(k_{\parallel}) = \frac{4\delta \sin[ik'_x d]}{\delta^2 \exp[-ik'_x d] + 4\exp[ik'_x d]}$$



Fourier spectrum $f(k_{\parallel})$



Reflection $|R(k_{\parallel})|$

Consider the small absorption limit

$$3).\ \lim \delta \to 0\ R(k_{\|} > \log|\delta|/d) \sim 2/\delta \to \infty$$

$$\lim_{\delta \to 0} \mathcal{I} \sim \begin{cases} C\delta^{\beta-1}|\log\delta| \to 0, & \beta > 1, \quad \text{imaging} \\ C'\frac{1}{\delta^{1-\beta}|\log\delta|} \to \infty, & \beta < 1, \quad \text{cloaking} \end{cases}, \qquad \beta = 2d_0/d.$$

As $\mathcal{I} \to \infty$

$$\phi^{\text{ref}} = \frac{\alpha\mathcal{I}\phi^{\text{inc}}}{1-\alpha\mathcal{I}} \to -\phi^{\text{inc}}, \quad p \to 0$$

In the no absorption limit, (-1,-1) slab is a perfect cloak for a small object inside a distance of D/2

The same physics in 3D

- Consider a point dipole in front of a slab with arbitrary value of ε and μ
- Calculate the effective polarizability

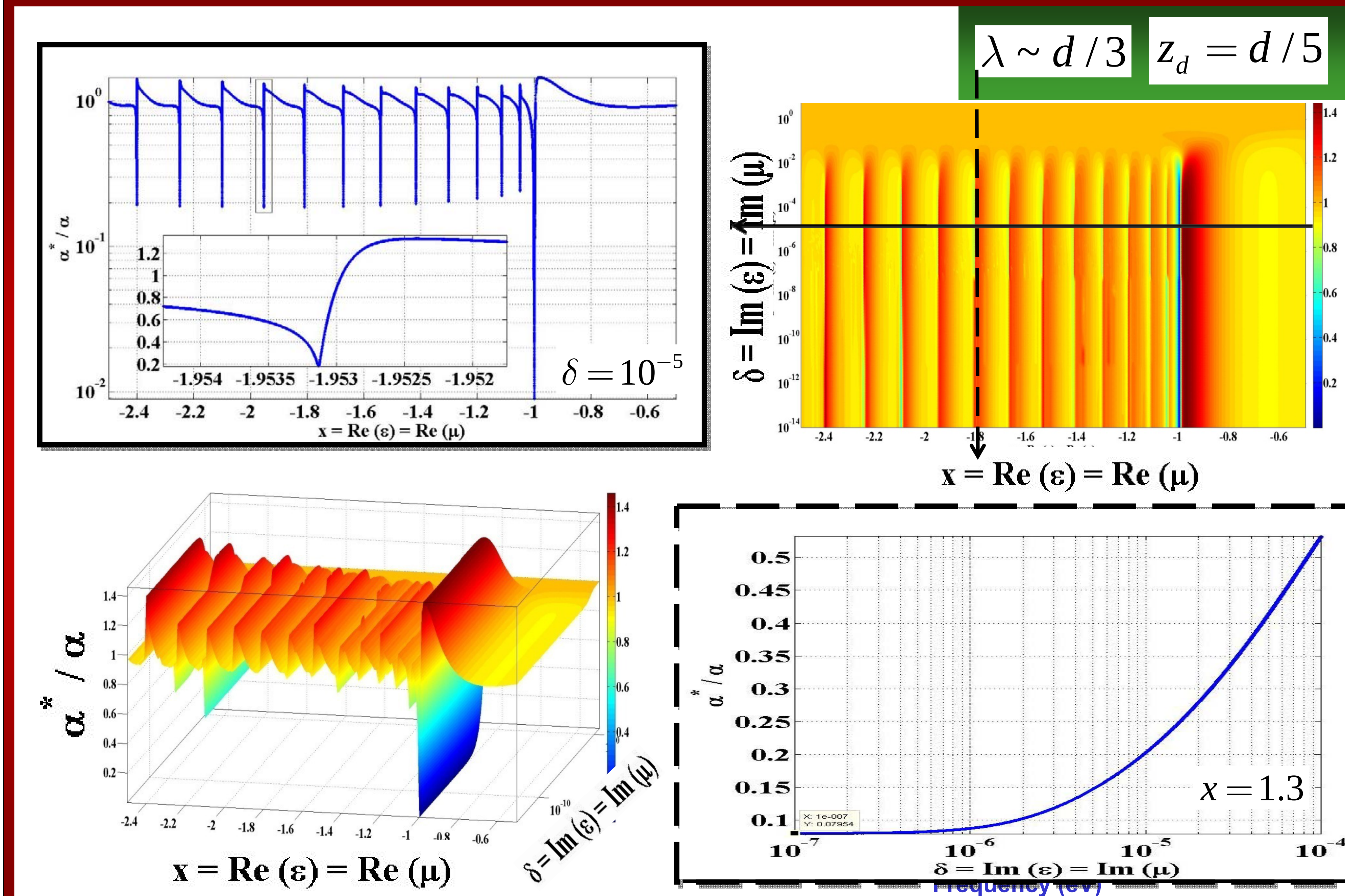
Effective polarizability of a point dipole

$$\alpha^* = \left[\alpha^{-1} - 4\pi k_0^2 W_{yy}^{ref}\left(\mathbf{R}_d, \mathbf{R}_d\right)\right]^{-1} \longrightarrow \frac{\alpha^*}{\alpha} = \frac{1}{1 - 4\pi k_0^2 \alpha W_{yy}^{ref}\left(\mathbf{R}_d, \mathbf{R}_d\right)}$$

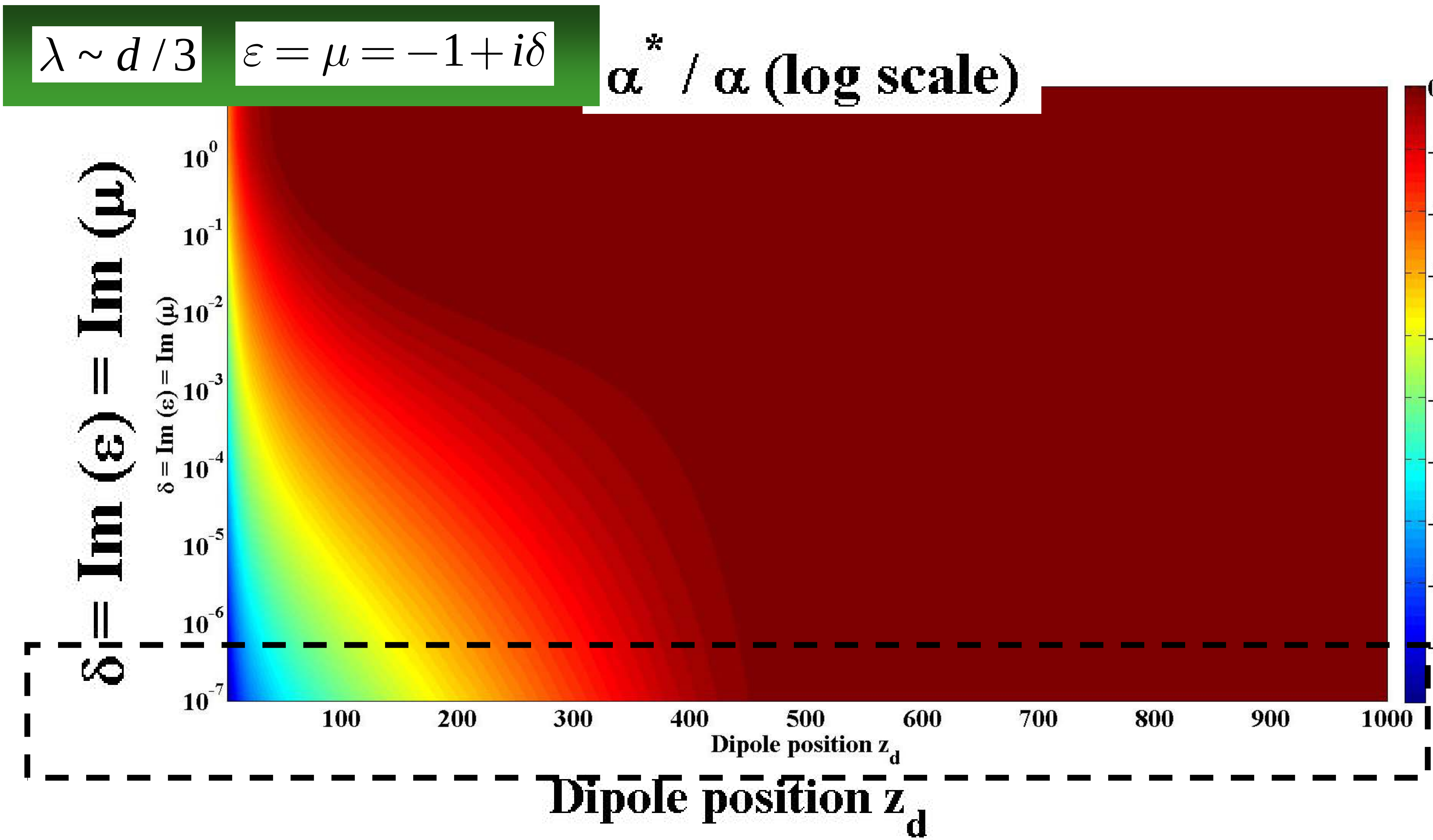
- **Four possibilities.**

1. $\frac{\alpha^*}{\alpha} \sim 1$ **: complementary media limit (perfect lens)**
2. $\frac{\alpha^*}{\alpha} \to 0$ **: perfect cloaking limit**
3. $\frac{\alpha^*}{\alpha} \in (0,1)$ **: partial cloaking, imperfect lens**
4. $\frac{\alpha^*}{\alpha} > 1$ **: resonance condition**

Dipole in front of slab



Varying z_d and delta=Im(eps)



Perfect cloaking | Partial cloaking | Complementary media

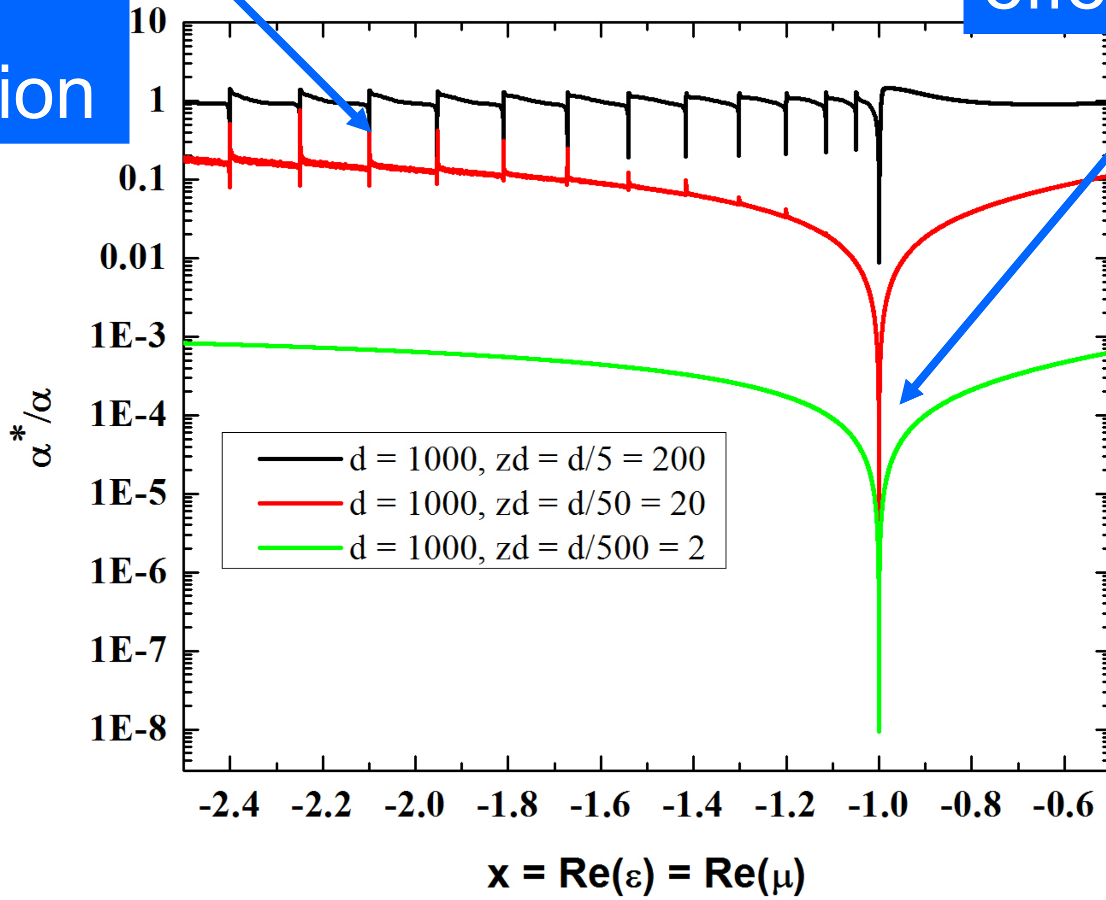
Two suppression mechanisms

Fabry-Perot effect

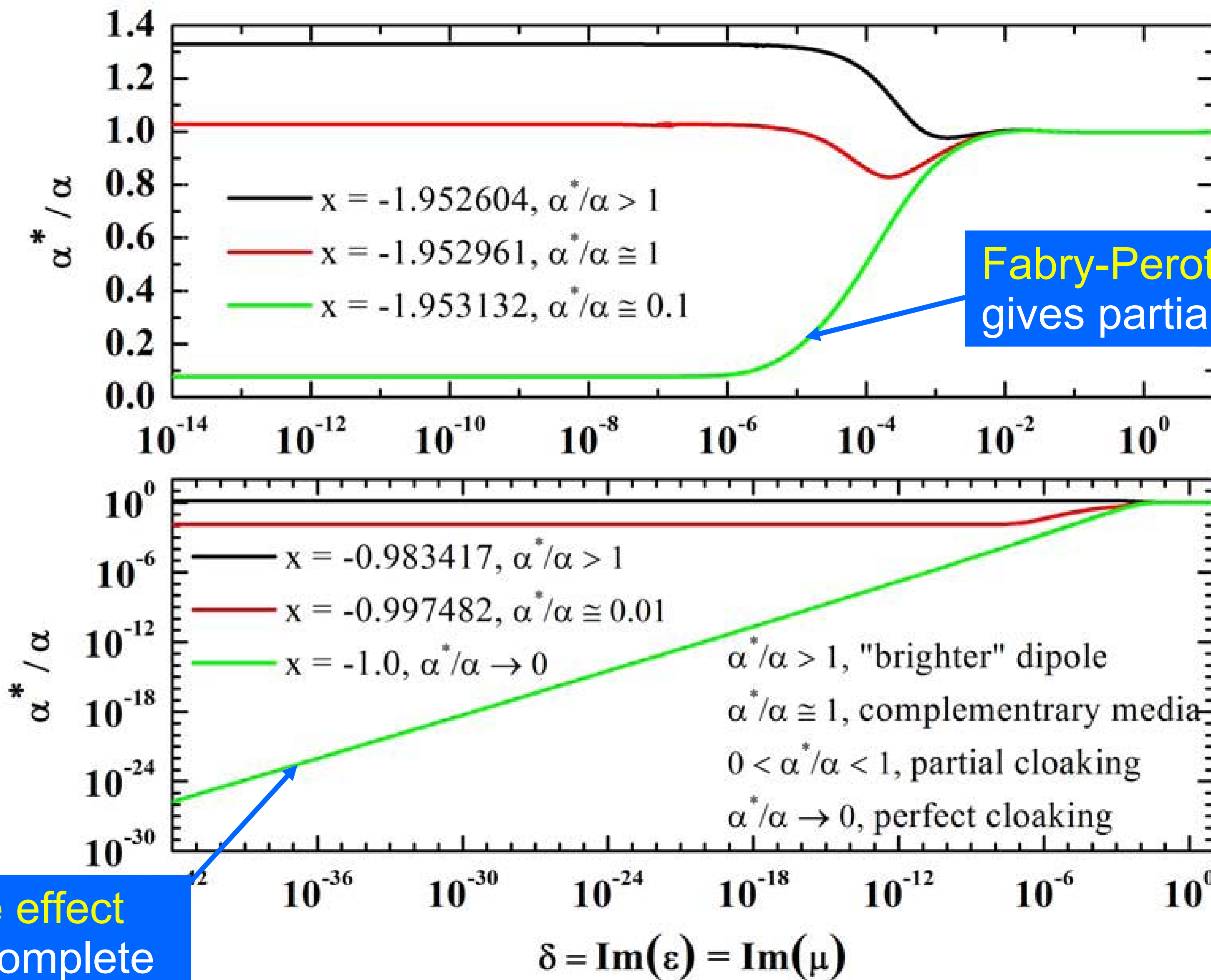
Not effective for thin slabs

Partial suppression

Surface resonance effect



Distance from slab = d/5



Fabry-Perot effect
gives partial suppression

surface effect
gives complete
suppression

Slab "cloaking"

- What if you push dipole closer to slab ?
  - Then dipole is always suppressed
  - Irrespective of the material property of the slab

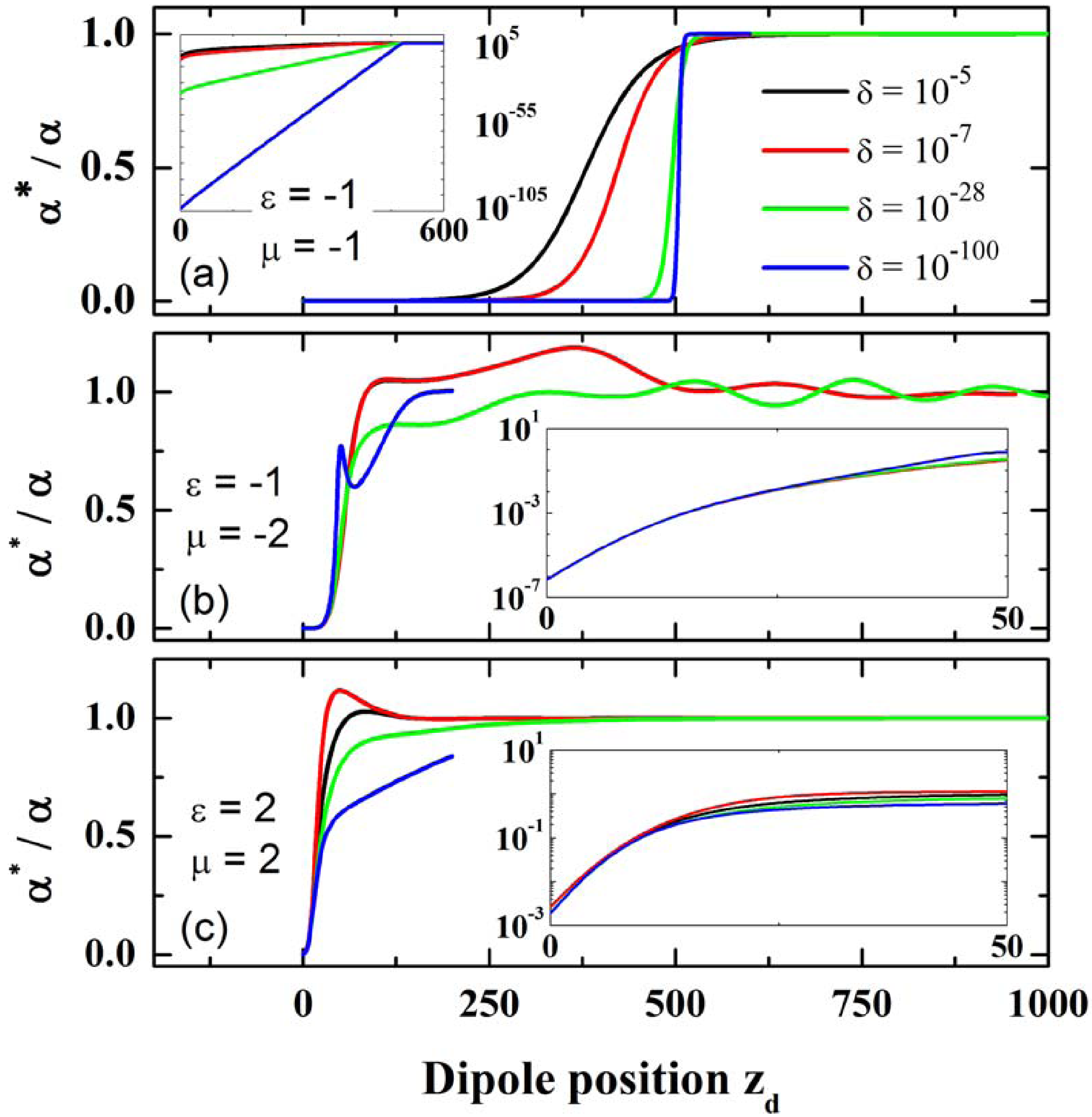
For a slab (ϵ, μ), If $k_{\|} > \frac{1}{d}|\log\frac{1-\mu}{1+\mu}|, \frac{1}{d}|\log\frac{1-\epsilon}{1+\epsilon}|$,

$$R \simeq \begin{cases} (\mu-1)/(\mu+1), & \text{for TM—}E_z \text{ mode} \\ (\epsilon-1)/(\epsilon+1), & \text{for TE—}H_z \text{ mode} \end{cases}$$

$$\mathcal{I} = \int_{-\infty}^{\infty} \frac{i\exp[2ik_x d_0]}{\sqrt{k_0^2 - k_{\|}^2}} R(k_{\|}) dk_{\|}\,.$$

is the same as $\int^{\infty} \exp[-\kappa d_0] \cdot \frac{1}{\kappa} d\kappa$, which diverges as $d_0 \rightarrow 0$

Any slab cloaks, but n=-1 cloaks better: it has a "quiet zone"



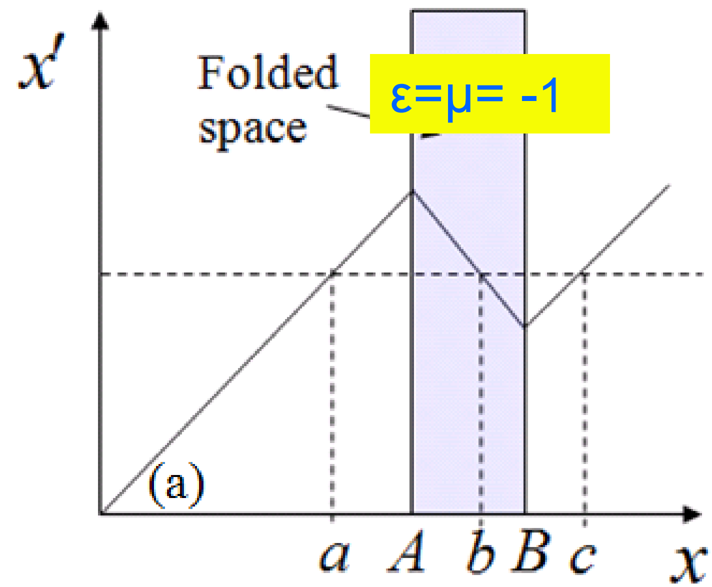
Slab "Cloaking"

- Any slab is a "cloak", in the sense that dipole excitation will be suppressed if dipole is placed close to the slab surface
- In general, the suppression is "smooth"
  - Induced dipole becomes smaller as it approaches the surface
- For $\varepsilon=\mu= -1$ ("super-lens"), the suppression is "abrupt":
  - Induced dipole $p= 0$ within a distance of $D/2$ in the limit of no absorption
  - i.e. it has a "quiet zone"

Is (ε=μ= -1) unique ?

- Is (ε=μ= -1) the only system that has a “quiet zone” ?
- The answer is NO, if we allow anisotropy

From the point of view of transformation optics, (ε=μ=-1) is the material corresponding to a “folding transformation” with dx’/dx = -1



"Folding transformation" media give cloaking with a finite quiet zone

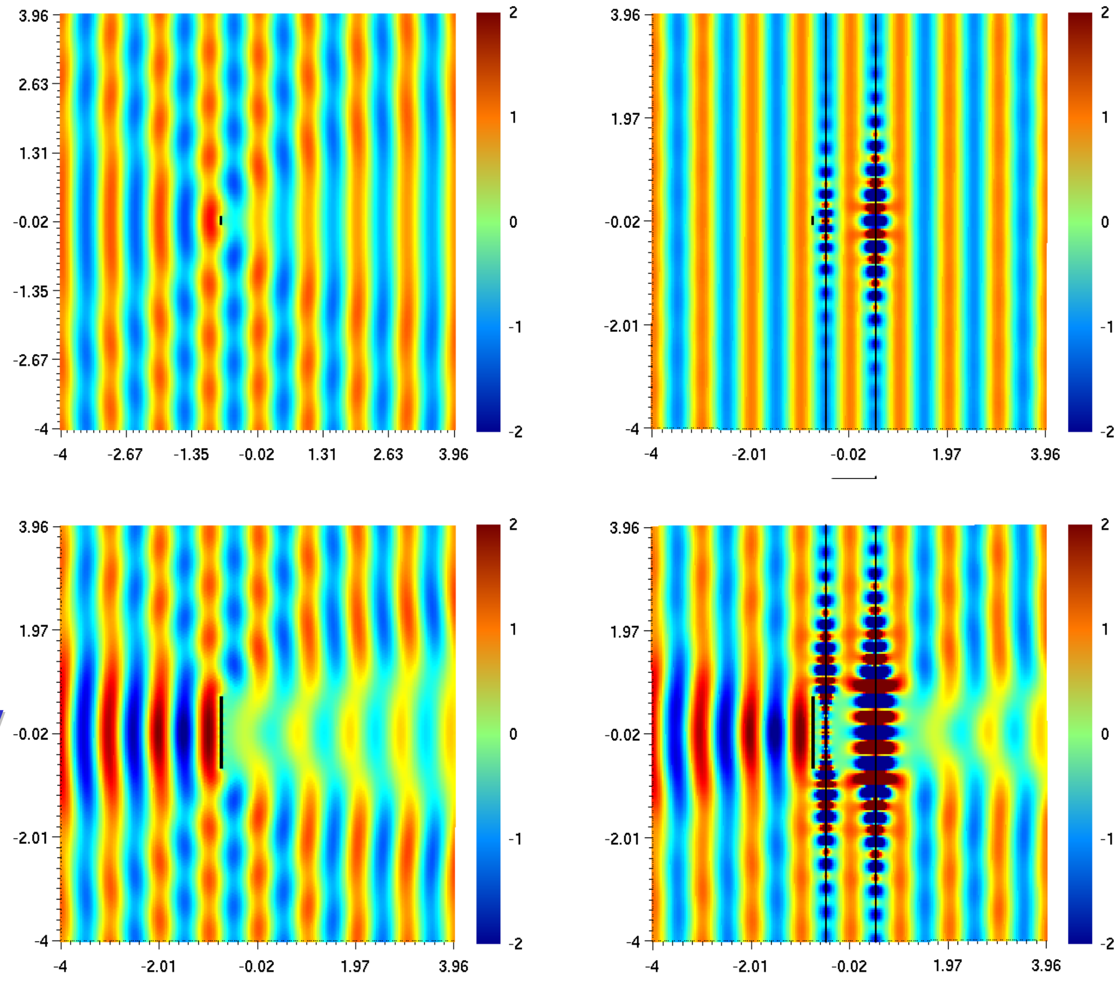
- A slab with $\varepsilon=\mu=$ diag$(-1/\beta,-\beta,-\beta)$, corresponds to a folding transformation with slope of $-\beta$, can totally suppress a dipole within a finite cloaking region
- The quiet zone is $\beta D/2$ (D=slab thickness)

Size effect - qualitative

Line: a X L
λ = D =1.0
a = 0.005 << λ
ε = 400
dis = 1/5 D < ½ D

L=0.1 << λ
Cloaking

L=1.5 ~ λ
Imaging
("complementary media")



without slab

with slab

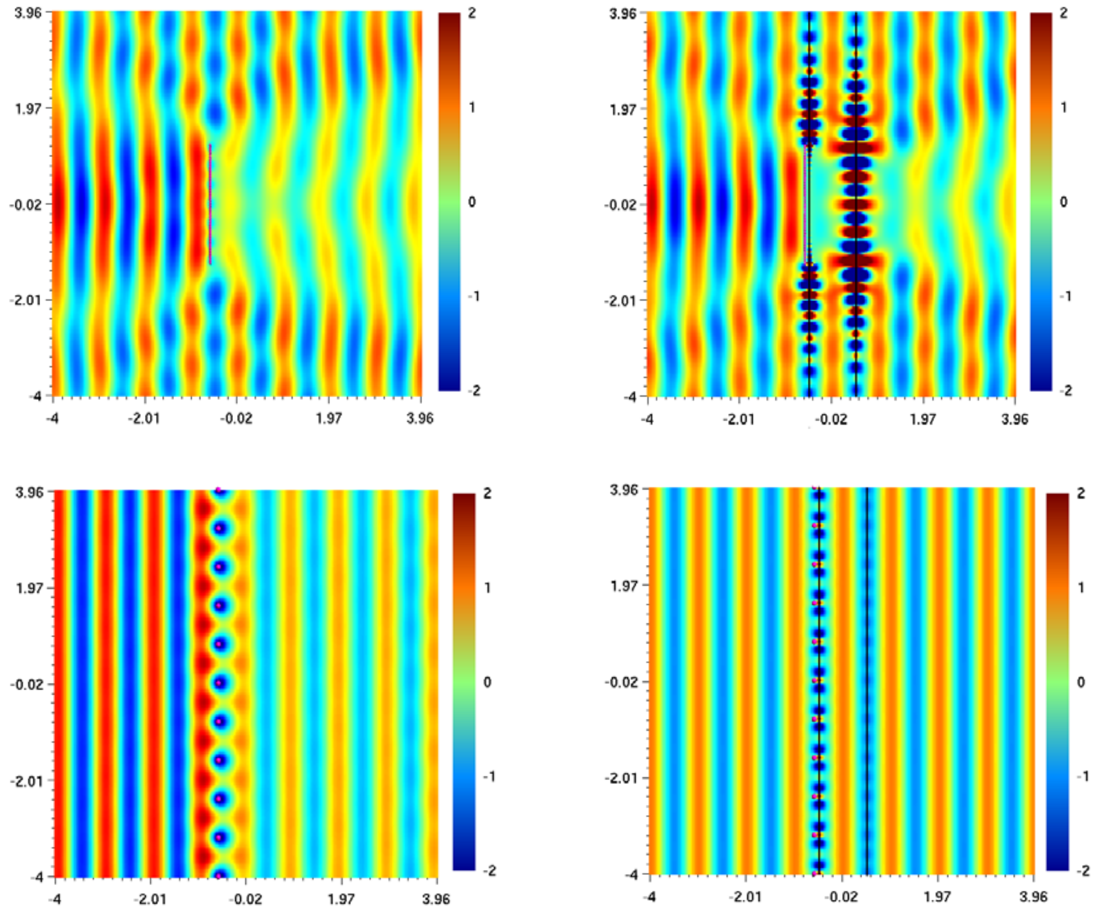
Cloaking or Imaging: depend on the size of the object(s).

Coupling between objects

λ = D =1.0
50 identical cylinders
Radius = 0.005
ε = 400
dis = 1/5 D < ½ D

Separation: 0.05<<λ

Separation: 0.8 ~λ



without slab

with slab

Coupling destroys cloaking

Conclusion

- A ε=μ=-1 slab can serve as a lens and also a cloak
- When absorption is not zero, it is a lens (not a perfect one)
- When absorption approaches zero, it becomes a perfect cloak for a "point" object (it is also a perfect lens, but the image has zero brightness)
- The cloaking effect also applies to other values of ε and μ. Those corresponding to a transformation media "folding transformation" has a "quiet zone"
- The slab cloaking is not effective for a big object, or a collective of small objects that are very close together

Collaborators

- JW Dong (SYS)
- HH Zheng (HKUST)
- Also
  - JJ Xiao, HH Zheng (BEM)
  - Lai Yun, Z Q Zhang, Kenyon Chen, Jack Ng (cloaking at a distance, illusion)

Introduction on rigorous Green function method

- Suppose an *active* dipole source ($\mathbf{p}_0$) acting on a *passive* dipole in front of a slab, its coupled dipole equation

$$\mathbf{p} = 4\pi k_0^2 \mathbf{M}^{-1} \overleftrightarrow{\mathbf{W}}^{00}\left(\mathbf{R}_d, \mathbf{0}\right)\mathbf{p}_0$$

Total Green function caused by the *active* dipole

where

$$\mathbf{M}^{-1} = \left[\alpha^{-1} - 4\pi k_0^2 \overleftrightarrow{\mathbf{W}}^{ref}\left(\mathbf{R}_d, \mathbf{R}_d\right)\right]^{-1}$$

Polarizability

Reflection Green function caused by the *passive* dipole

Introduction on rigorous Green function method

- **Compare with a particle in free space**

With the slab

$$\mathbf{p} = 4\pi k_0^2 \left[\alpha^{-1} - 4\pi k_0^2 \overleftrightarrow{\mathbf{W}}^{ref}\left(\mathbf{R}_d, \mathbf{R}_d\right)\right]^{-1} \overleftrightarrow{\mathbf{W}}^{00}\left(\mathbf{R}_d, \mathbf{0}\right)\mathbf{p}_0$$

Without the slab

$$\mathbf{p} = 4\pi k_0^2 \left[\alpha^{-1}\right]^{-1} \overleftrightarrow{\mathbf{W}}^{00}\left(\mathbf{R}_d, \mathbf{0}\right)\mathbf{p}_0$$

- **We can define an effective polarizability to describe the dipole response of the system. For TE wave,**

$$\alpha^* = \left[\alpha^{-1} - 4\pi k_0^2 W_{yy}^{ref}\left(\mathbf{R}_d, \mathbf{R}_d\right)\right]^{-1}$$